

HOW FUTURE TEACHERS IMPROVE EPISTEMIC QUALITY OF THEIR OWN MATHEMATICAL PRACTICES

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In this paper we present how the future mathematic teachers improve their self-reflection in didactical analysis competency. We draw on data collected from 2 groups of prospective teachers, using qualitative methods. We use a prospective teacher as an example to discuss how the training on the use of didactical suitability criteria and reflective tools lead them to learn about the experience, and explain in a deep way how they will change their own practices in the future.

Keywords: *mathematical quality, professional development*

INTRODUCTION

In this paper we analyze how a specific pre-service mathematics teachers' training program produces changes in terms of future secondary school teachers' competence of didactical analysis. In particular, aiming at the growing and building mathematical knowledge for teaching (Zavlaswski & Sullivan, 2011) by using theoretical tools for self-analysis. Our general intention in such a program is to lead future teachers to develop the professional ability of reflecting about changing their own planning by using didactical analysis theoretical tools. In this paper, we focus on explaining how future teachers value the need of analyzing mathematical quality in their final reflections when they tell us how and why they will improve their school practice for the next future. The research relates to the dilemmas of task redesign that provides a framework that can be used for analysis of quality and suitability of tasks (Giménez, Font & Vanegas, 2013). Our aim in this presentation is *to describe how a specific set of professional tasks, promotes the emergence of using theoretical tools from future teacher's reflections, when valuing epistemic/mathematical suitability of an instructive process in a multidimensional way.*

In our study we call 'professional task' those tasks that we propose to future teachers in order to encourage them doing didactic analysis and developing their didactical analysis competences. We want to focus on some immediate effects of the task redesign done by the future teacher. In previous researches (Vanegas, et al., 2014), it was found it when analyzing prospective teachers' thoughts emerging from their feedback [work assignments] with the researchers; and also emerging from our analysis of some impacts of the program itself. Such above mentioned development, it is stated when future teachers incorporate and use tools for the description,

explanation and process valuation of mathematical school teacher/learning practices. Thus, our main hypothesis is that future teachers can reflect and value their own practices in a deep way, and to have ideas to structure a possible redesign that improves the quality of their own mathematics and teaching for the future.

THEORETICAL FRAMEWORK

In our research, we assume a learning through teaching perspective between the design of classroom tasks, the pedagogies associated with the effective implementation of tasks and the learning of mathematics (Leikin, 2006) and task design problematics. The role of the Mathematic teacher in such a framework is to select, modify, design, redesign sequences, implement and evaluate mathematical practices. The analysis and description of the mathematical activity is conducted using the theoretical constructs proposed by the 'Ontosemiotic' approach (OSA). According to this perspective (Godino, Batanero & Font, 2007), the mathematical activity plays a central role and it is modeled in terms of systems of operative and discursive practices. From these practices the different types of related mathematical objects emerge building cognitive or epistemic configurations among them. Problem situations promote and contextualize the activity; languages (symbols, notations, and graphics) represent the other entities and serve as tools for action; arguments justify the procedures and propositions that relate the concepts. Lastly, the objects that appear in mathematical practices and those which emerge from these practices might be considered from the five facets of dual dimensions (Godino, Batanero & Font, 2007). Both the dualities and objects can be analyzed from a process-product perspective, a kind of analysis that leads us to the processes shown.

In fact, there are two possible perspectives to analyze the quality of a mathematical practice. On the one hand, international experiences focusing on rigor and mathematical richness (Hill, et al., 2008). They introduced a set of categories to measure the mathematical quality of an instructive process (Hill, 2010): (1) well done format, (2) work done in a connected way to mathematics, (3) richness of the mathematics itself, (e) work done with the students (4) errors and imprecise language used, (5) students' participation. It's also explained that for mathematical richness, we can see some indicators as: mathematics explanations, multiple procedures or solving methods, mathematical generalization development, use of a mathematical language, and general mathematical aspects of richness as representations, among others. On the other hand, by considering OSA we interpret that it is necessary to see the theoretical tools not as a priori solution, but an emerging process from the practice itself. We consider that it relates to a way of constructing quality criteria by the future teachers during the professional training process. Hypothetical trajectories should reveal not only the acquisition of theoretical tools for analysis, but also to see how personal didactical principles are assumed by future teachers in an open and personal way. The notion of epistemic quality proposed by OSA is centered on

representativeness of mathematics taught by the future teachers in a lifelong learning perspective according a holistic meaning of mathematical objects to be learnt, understood as a pair (mathematical practices, primary objects and processes activated in such practices). The determination of such a global meaning or holistic meaning requires an epistemological/historical study about the origin and evolution of the mathematical object. We must take also into account the diversity of “using contexts” in which it’s possible to play each of the possible configurations of these primary objects.

There is a consensus on considering what epistemic suitability mean: Representative and articulated sample of diverse types of problems (contextualized, according different levels of difficulties, etc.); use of different modes of expression (verbal, graphic, symbolic...), also with translations among representations. In such a way we could consider an adequate level of mathematical language; definitions and procedures should be correctly stated and adapted to the level of the students; presentation of basic statements of the “topic” in order to, establishing relations and meaningful connections among definitions, properties, problems and so on. The complexity of perspectives about mathematical objects from OSA perspective, introduces the idea that a mathematical object is not simple but a complex system. The idea of representativeness as quality criteria is related to coherence and connectness aspects.

In a previous research, it was observed that epistemic suitability criteria and epistemic configurations, could be useful tool to organize self- reflection, but those tools were not used by future teachers in a pre-implementation as planning phase (Giménez, et al., 2012). Now, our research aim is to confirm if the future teachers could emerge the suitability criteria from their final analysis. We also want to see how they use it, by describing a future teacher case study.

METHODOLOGY

According to the proposed aim for this study: a) a set of professional tasks had been designed; b) these tasks are implemented in the training program; c) the written productions of future teachers are considered as essential data and d) a student production is analyzed as a case study.

We assume a learning by teaching approach based on an inquiry and reflective practicing in which it was observed the tensions presented for task designers and teacher adaptation whether they are designing tasks for themselves or for others. Framework in which we design and implement diverse teacher training cycles as teaching experiments for developing didactical analysis competences among others. In particular, we discuss some effects of a teacher training cycle named "Epistemic Analysis" (as a part of a more general cycle of didactic analysis). The development

of this cycle includes four main professional tasks: (a) First naïf analysis of a teaching episode about proportional reasoning, and refined analysis using suitability criteria (b) Looking for errors, ambiguities, and construction of different meanings by analyzing three episodes about perpendicular bisector concept; (c) Analysis of connectness and representativeness; (d) The need of improving richness of processes, by analyzing episodes of introducing integral concept. After that, the future teachers designed and implemented a lesson into their period of internship. They did a first reflection and valuation of the suitability from their didactic implemented unit. Finally, we ask for an improvement and re-planning of their lessons designs (for future implementation), within the Master's Final Project (MFP).

As a general schematic framework, we see in figure 1 the developmental process.

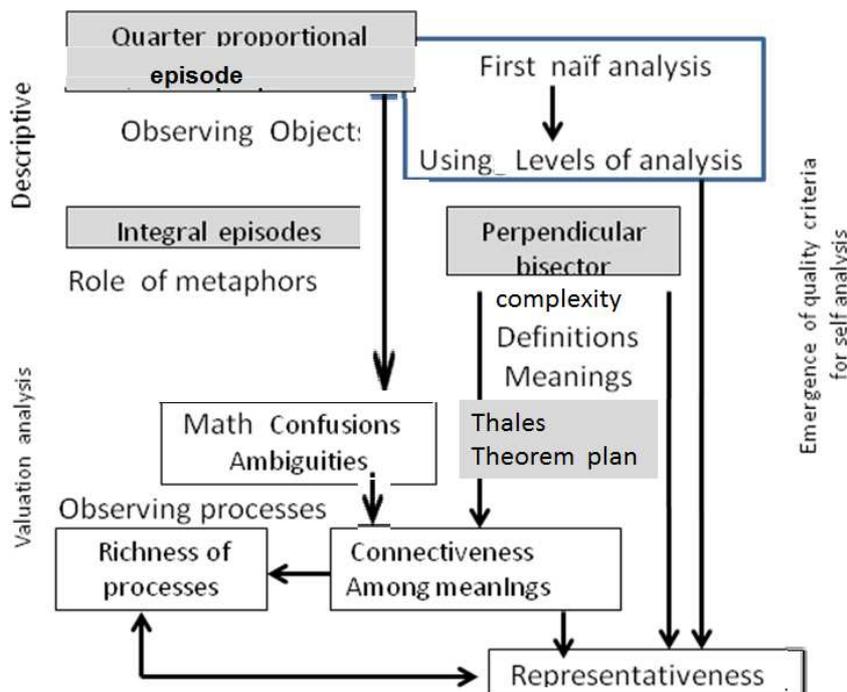


Figure 1. Scheme of different constructs involved in epistemic cycle

During the cycle we present theoretical tools (suitability criteria, according Godino, Batanero & Font 2007) to conduct evaluative analysis to answer “what could we improve from our self-analysis?” We understand that the study of descriptive and explanatory analysis for a didactical situation is necessary to justify the evaluations. As the aim is to give tools for reflection, we use methodology based on case study design based research (Gravemeijer, 1998): the use of a real environment, to generate new and efficacy learning environments, collaborative enquiry perspective among trainer-researcher and future teachers, searching simultaneously construction of theories and practice innovation (Cobb, di Sessa, Lehrer y Schauble, 2003).

The study was conducted by two groups of 32 future Secondary School teachers of Mathematics from the Interuniversity Program in Catalonia. For the results, we analyze the final work of a single student in terms of his use of tools. The data were

registers on virtual platform, audiovisual comments about classroom episodes, final Master's work valuing mathematical quality about their implemented own practice. Now, we explain each of the analysis through the four main activities introduced.

(a) Task 1. Noticing mathematical objects and processes in a math task. A first task had been introduced to show how constructs emerge from a school practice as cognitive and semiotic conflicts, epistemic obstacles, types of norms, interactive, patterns of models of management, etc. It was selected an initial task in which students confront a short case study about proportional reasoning, using transcripts of a classroom situation. Such an initial *tasks* introduce the students for reading and analyzing the classroom example, by using their previous knowledge and beliefs of didactic analysis. During the first task, future teachers did naïf comments about a proportion class talking about the comparison of two quarters with different density of population. It's easy for them to identify mathematical objects but it's difficult for them to recognize all the processes involved in the task.

Refining the analysis by using theoretical tools. It was analyzed a class about equations by applying structured suitability criteria. The students start by analyzing mathematical practices, by observing objects and processes. Then the trainer develops an example in which it was revised suitability construct. After that the future teachers reflect, improve and refined their analysis by using the notion of epistemic suitability. In such professional task, it's still difficult for the future teachers to identify some semiotic conflicts. Later, in the different subjects of the Master, the students realize other analysis of practices, objects activated in the above mentioned professional practices (problem, definition, proposition, representation and argument) and mathematical processes. Observing such initial analysis realized by the future teachers, we expect to improve some difficulties to distinguish between concepts and definitions. It's also found that during second analysis, the future teachers recognize their duplicities between definitions, propositions and procedures, and also their duplicities between propositions and thesis of arguments. It's still difficult for them to have a good description of practices because they overlap the configuration of objects and the description of processes. It's also difficult to see and to catalogue mathematical processes. The reflective process in this training period, try to add historical values of mathematical objects, relations among concepts, and to analyze how the use of resources can change the construction of different meanings. Therefore, it is expected to see mathematical structure behind the mathematical practice.

(b) Task 2. Analyzing the role of mathematical errors and ambiguities. Initially, the first proportion professional task was enough to see that if you did or introduced misconceptions or errors you will promote semiotic conflicts. It's common sense of quality the idea of coherence when using mathematical errors. But another task was introduced to review such idea about perpendicular bisector.

Analyzing the role of definitions and meanings. In such epistemic analysis, we notice that it's needed an adequate contextualization, in order to delete abuses, to focus on mathematical purposes, and to promote transference of knowledge. As a prototyping professional task to promote such analysis, it was proposed the observation of three short ways of introducing perpendicular bisector with 12-13 years old students, by observing three different teachers. The main idea to present a discussion about the different practices, objects and mathematics processes and to introduce a reflection associated to how each of these episodes contribute to introduce different kind of epistemic configurations and objects associated to three different definitions. First practice introduces perpendicular by the middle point of the segment. Second teacher use a definition as right-lane formed by the points being equal distance from the end points of the segment. Third teacher uses the idea of border of two regions determined by the closest neighbour principle) It was observed that both first and second teachers did classical proposals and management about the content and the classroom. The third teacher proposal is innovative not only because of the management but mathematically as a way of changing the regular use of mathematical content as a change of configuration of practices, objects and mathematical processes by using a non-routine task. Future teachers observed interpretation processes, communication of didactical and mathematical meanings, etc. Furthermore it appears a reflection about distinguishing complex processes from simple processes and also a general reflection about the idea of processes itself.

(c) Task 3. Analysis of connectness and representativeness of knowledge. To characterize epistemic quality, we analyze how representative and articulated set of activities/problems (contextualized, different levels of difficulty, different modes of representations and translations, etc.) had been proposed. It as a way to see the need of clarification of questioning and exemplifying, because in such situations we can see how different meanings appear, how we establish relations and connections among definitions, properties, and problems.

A prototypical example of this task analysis is a case based analysis upon a previous future teacher reflection that: first planned a sequence with 7th grade (13-14 years old students) for Thales theorem. She analyzed her own practice about Thales Theorem after the school practice, but she didn't notice a good mathematical connection among different activities. The trainer introduces epistemic configurations to see that some connections had been forgotten. The aim of this professional task is to recognize a deep level of analysis from such previous prospective teacher's practices (Choppin, 2011). Thus, the future teachers learn from this analysis, the need of connecting several epistemic configurations.

(e) Task 4. Improving richness of processes. It was decided to use the integral concept to start such a general analysis. The episodes presented come from a school experience in which a substitute teacher uses her common knowledge with a regular textbook to introduce integrals for 18 years old students. The trainer explains the

idea of richness of processes, and talk with future teachers about the complexity of the integral concept by noticing seven meanings as geometric, result of a change process, inverse of a derivative, limit approach, generalized content, algebraic, and numerical method (Contreras, Ordóñez y Wilhelmi, 2010). After that, it was questioned which processes they think it's possible to promote by using some problems. We considered if the situations activate some of the following processes: contextualization, algorithmisation, communication, argumentation and problem solving. In such analysis it's also discussed the relevance of some specific questions. After such task analysis, there was a consensus about the low level of accuracy in terms of promoting processes associated to different meanings.

RESULTS AND DISCUSSION

In order to reflect about how the future teachers introduce and interesting self-reflection, we exemplify some important unexpected acquisitions as a special case study to understand possible influences of the above professional tasks upon their epistemic valuation. The future teacher (N from now), was considered interesting for being analyzed because he is not mathematician, but economist, and he uses very carefully the theoretical framework above presented for his own reflection. Some teachers tell us that they didn't introduce mathematical errors. It is important to say that in previous years, they were not worried about it.

“I did an error introducing the notion of compatible undetermined system, by using an apparent good example of prices, but telling the students that there are infinite solutions. It was difficult to see, but I promoted an error, because the contextual situation is related to a finite set of answers. The children conclude that there are many solutions, but limited because of non existence of negative money. Nevertheless, I didn't say anything about the limited use of coins in Spanish currency”. (N)

The same teacher explains explicitly how he will change a statement in a problem, because of the reflection.

The future teachers accept that they did ambiguities relating language problems that they identify as creating possible semiotic conflicts.

I promote some statements, not enough clear as we see in the task ... Finally, my tutor tell me about the need for clarifying why we used the expressions “It passes subtracting” “We delete denominators ... and others”. (N)

According formalism introduced, the future teacher assumes that he started with not necessary sentences. The teacher T relates such ambiguity to a theoretical article. For instance, he said about “*the need of searching analogies found because of an incorrect use of contextual framework*”. He read a text from Reed to reflect about the use of two important variables influencing the decisions of the teacher.

“The context understood as a set of traits perceived in a certain problematic of real world involving objects, and facts”... But, the laws, principles, relations among quantities, and equations, constitute the structure of a problem”. It is interesting that the future teacher explain some conclusions from this discussion: “the need to describe the similarities and differences among structures and surfaces of the source problems and aiming problems, because it influences the decisions about the equations presented to solve the aiming problems. It is also important to identify that familiarity can help the transference processes, but it also could be an obstacle to see the similarities and structural differences among problems”. (N)

About richness of processes. The future teacher expressed the need to incorporate problem solving from the perspective of Polya, which had not been considered in his planning. He also explains the need to articulate the role of letters and unknowns.

It would be necessary to change the status of quantities designed by letters We must identify the global traits of mathematical competency, not only because of my experience, but the studies conducted in which it's found the serious difficulties to produce right equations. If it's correct, is because of the use of Cartesian methods in a flexible way, producing a diversity of equalities for the same problem... It's also important to identify different patterns when we build equalities. (N)

We also found in his work, much unexpected reflections about the need of modeling processes in an inquiry perspective. The future teacher N tells about “*the emphasis on algorithms instead of modeling that gives opportunities to observe similar structures in the same model...*” Quoting Chevallard “*presented three important steps for modeling in algebraic situations: To introduce letters to define variables of the system, that gives opportunities for generalization and increase mathematical power. To establish relations among variables and to work mathematically to establish new relations....*” The future teacher N, not only recognize the students' difficulties but also indicate that “*some researches explain the possibility to validate the model and learning from such a perspective ... designing courses using graphic calculators*”.

About representativeness. The future teacher tells us that it's needed not only a look for meanings, but to see a historical, epistemological and curricular perspective. Then proposed a set of ideas about the Arabic way of solving problems to be introduced next time. In this case, he just offer a reflection about “*considering algebra as part of cultural legacy*”.

About connectiveness. N explains the need for applications to other disciplines.

As an economist, I can say the use of systems of equations to find equilibrium points, as intersection of different conditions, interpreted by curves of offer and demand... and planning problems of dead points... programming problems... We also use algebra as a

process to solve engineering problems... chemistry problems,...restoring digital images... About intramathematical connections, algebraic systems of equations are a referent knowledge for optimization problems...We assume that some of these knowledge must be introduced and adapt according the age of the students. (N)

CONCLUSIONS

The professional tasks proposed, promote that the future teachers did positive reflections about their own practices. In each set of tasks and analysis we could identify the emergency of the different aspects characterising the epistemic suitability. Some of these are in agreement to which were presented by Wilson, Cooney & Stinson (2005).

The different type of analysis done during the implemented Program, not only gave opportunities for establishing categories and structure for the reflection, but also permitted to explicit how to improve the didactical sequence as we have seen in the case study. We consider that such attitude for changing from reflective analysis is important for professional development.

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