Acevedo, J.; Font, V.; Giménez, J. (2004) Globalisation and mathematics Education CIEAEM54 (pp. 336-342)

Class Phenomena related with the use of metaphors, the case of the graph of functions

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ABSTRACT

In the presentation we focused, from an enactivistic perspective, the effects that produce the uses of metaphors in the analytic study of graphical of functions in High Secondary School (16-17 years old students). The central metaphor *"the graph of a function can be considered as the outline that leaves a point that moves subject to certain conditions" is speciffically observed.*

RESUMÉ

Dans cette présentation, on analyse, du point de vue enactiviste, l'effet produit pour l'usage des metaphores dans les classes sur l'étude analytique des graphes de fonctions à l'Ecole Secondarie Supérieure (16-17 ans). Le travail se centre sur la metaphore "le graphe de la fonction pourrait être consideré come la trace d'un point qui se déplace sur certaines conditions.

INTRODUCTION

Since Lakoff and the Johnson (1991), showed the importance of the metaphoric thought, understood as the interpretation of a proving field in terms of another already well-known, the role of the metaphoric thought in the formation of the mathematical concepts, has more relevance in the investigation in mathematics education (v.g. English 1997, Font 2000, Lakoff and Nuñez 2000, Nuñez 2000, Pimm 1990, Van Dormolen 1991). As it the cognitive theory of the mathematics of Lakoff and Nuñez suggests (2000) the mathematical concepts can be understood based on the relation of our body with their daily surroundings, as much as sociocultural as physical, thanks to the role that metaphors plays as a bridge between our body and our mind. In other words embodied mathematical concepts.

The presented investigation assumes enactivistic perspective, and deals with the phenomena related to the use of metaphors on the side of the teacher when explaining the graphical representation of functions with 16-17 years old students. The results of the investigation are related to the following key questions: (a) If abstract concepts are metaphorical as it's suggested by the enactivistic perspective, what are the metaphors used in the production, systematisation and communication of mathematical thinking?

And (b) New technologies provide the possibility of making more experiences. Can this help the development of more powerful metaphors aimed at constructing, organising and communicating mathematical thinking?

The objective of the research was to verify if these metaphoric phenomena can be considered like a didactical phenomena with a certain generality. Concretely we considered the following subobjectives: (1) Detect if the professor uses metaphors of the kind of "the graph of a function can be considered as the outline that leaves a point wich moves subject to certain conditions. Detect, in addition to the previous metaphor, other metaphors in its speech. (2) Verify if teachers are conscious of their use, and how they control it didactically. (3) Determine the effect that these metaphors produce on the students.

RATIONALE

In order to find the metaphors produced, some experiences were developed that were analyzed from a interpretative point of view. The investigation tried to understand the events so that they were interpreted the subjects investigated by means of an immersion in his thought and its practice. The data were obtained observing and recording different classes given by different professors in catalan secondary schools, with 16-17 years old students. An epistemological-historical analysis shows that the graphs had been structured in general from the following metaphors: (1) the curves are sections, (2) the curves are the outline that leaves a point that moves into certain conditions (3) the curves are the outline that leaves a point that moves into certain conditions and the analysis of these conditions allows to find an equation that fulfils the points of the curve and (4) the graph of a function f is the set formed by the points of coordinates (x, f (x)).

A case study was speciffically conducted in the study of analytic view of functions. In the analysis, we centered in the effects that produce over the education of the graphical representation of functions in High Secondary School, the use of the metaphor "*the graph of a function can be considered as the outline that leaves a point that moves subject to certain conditions*". - or a variation of this metaphor: *"the graph of a function can be considered as the outline that a point that moves leaves on the graph"* . - (observed by Font, 2000). In the following example, we analyze some trends of a part explanation about representation of functions done by the teacher .observing which

properties determine the "shape" of a graph. In particular, it's usual to say "a decreasing function is stablished if first derivative is negative. On the other hand, many teachers reduce these explanations to a 40 minutes lesson.

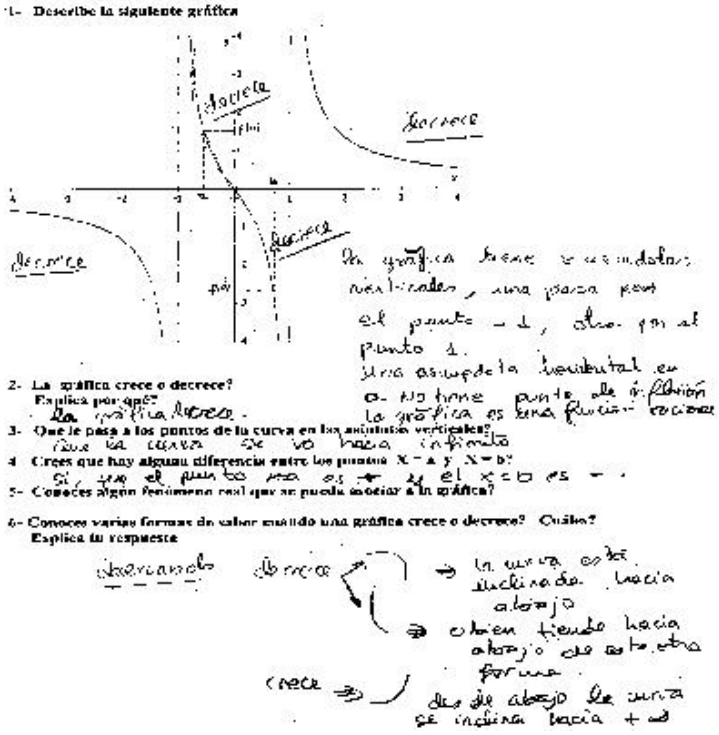
Teacher's speech in the classroom	Comments
Then we see that the degree of the denominator is greater than the degree of the numerator, then automatically we can say. Dominion.....	He writes the function **$y=x/(x^2+1)$** and starts to develop his analitic study
The denominator will grow much more quickly who the numerator..... Look , please.… .	It indicates with gestures the slate
We had been saying that if the degree of the denominator is superior to the degree of the numerator, when x becomes great.....crece much more fast the denominator who the numerator, therefore will be much greater, will be much greater the denominator as the x grows, that is taking great values.....	He asks to the students who make less noise
The denominator will be much greater than the numerator.....	Teacher moves his hands treating to reinforce the idea, shows how the x moves
Every time will become smaller.	Continuous moving the hands simulating the growth of the x and trying to show that the x can be returned greater

Teacher introduced numbers as objects that increased and it seems that they can be enlarged. He uses sentences as "denominator increases more quickly than …", "x becomes greater..". These metaphors can be considered as a grounding ones, based out of mathematics. It seems that the teacher needs to express this dinamic visualization to have an interpretation of the mathematical concepts he had been introduced. After analyzing such discourses, semistructured questionnaires "ad hoc" and interviews were conducted in order to analyze the discourse. Let's present a part of a discourse teacher with 15 years old experience .

transcription	Research comments
Interviewer- I observed in the video that you tell sentences as “here itcomes in such a way, it comes by here, ...numerator increases more than denominator. And when you speak, you indicate by raising hands, and doing movements... what do you want to express by means of it?	Trial for interpreting the awareness of the situation and the role of gestures

Teacher- mmm!Well, I imagine that it's completely inconsciencious, because I didn't think I want to say this or that, and I will move hands in such a way like this... What the gestures are making explicit? I believe on gestures as a reinforcement of the expressing ideas. Isn't it? Telling "it comes by here" I really should indicate that you follow the fonction, in the sense where the function goes on...	Assuming the reinforcement, the teacher revealed not to be aware on the difficulty involved.

To reflect about the impact on students' learning, two questionnaires were also prepared. In the figure, we show the example of one question in which is proposed to analyze the function by means of its graph.

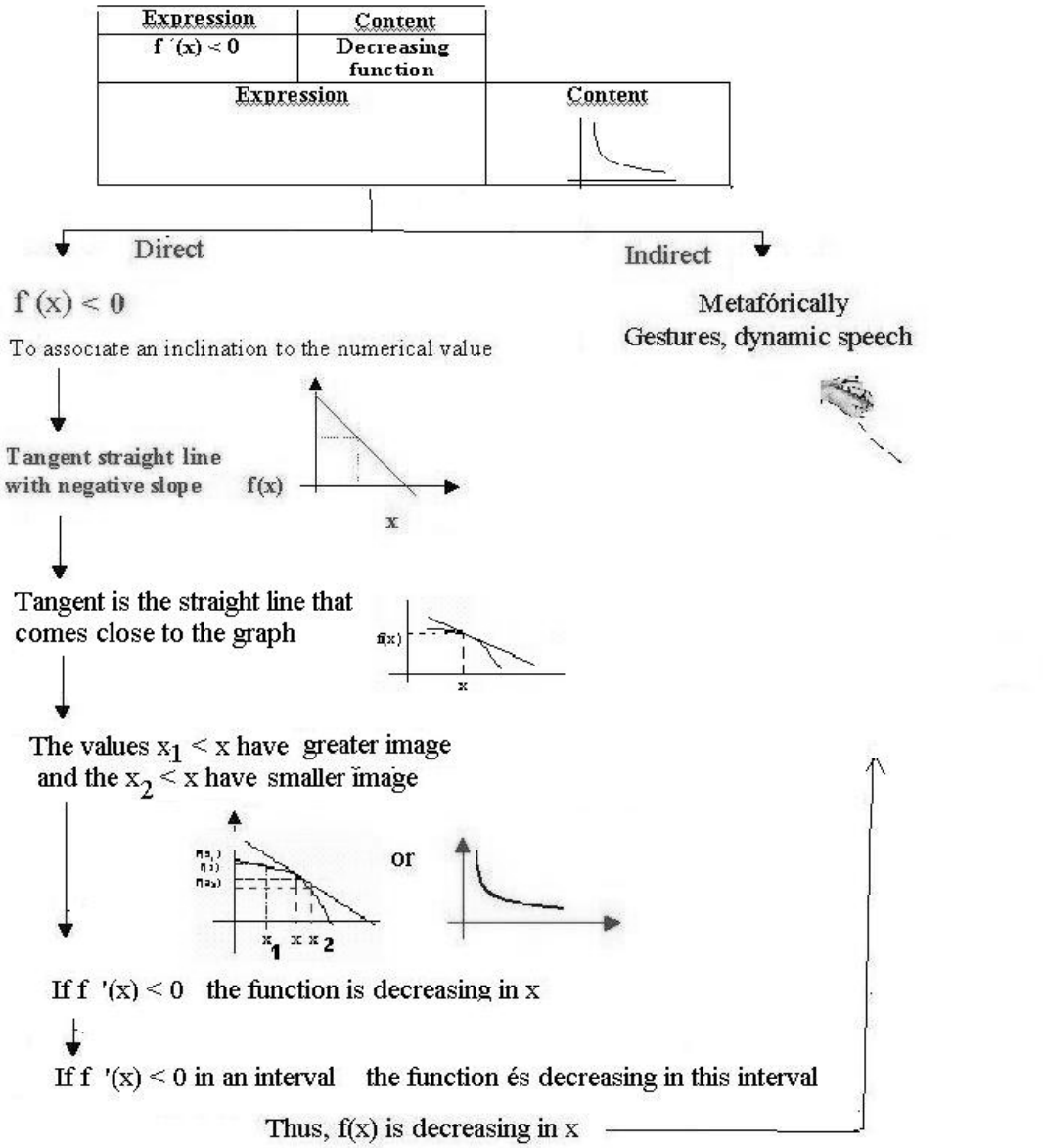


About the student comment. The function decreases in" such a way" (following the shape of the hyperbola instead of revealing the revealed sentences in which they said: "The function decrease means that has this shape" in algebraic relation, and derivative)".

The different answers showed that the students have been anchoraging on the visualized dinamic metaphor. Students in fact, accept litterally the gestures and speech of the teacher, and the gestures of the professor entail -although it is of unconscious way- to the use of dynamic metaphors. The teacher doesn't have enough "didactic control" on the situation.

Analyzing the classroom in terms of semiotic functions (following Godino & Recio 1998, Font 2000), we observe that the teacher could express all the steps and techniques

necessary to relate cognitive facts, but he usually decides to go directely to a metaphoric presentation, in which many mathematical aspects are deleted. Need of time is the usual reason intr oduced by the teacher to justify his decision. Let's observe the set of relationships in the exemple above cited. In the following diagram we can see how the teacher delete a part of the reasoning



In fact, the mathematical facts and techniques involved statically, are presented dinamically. This strong decision, motivated by the short time devoted usually to this topic, could be a possible explanation to interpret the students' difficulties and interpretation.

In a computer environment, we also observed that the use of computer graphers (calculating grapher, "calculate", Cabri...) entails to understand the graph like the trajectory of a point, which implies to structure the concept of graph in kinetic terms, and is reflected in the used language "**point that moves".** Graphs are identified as designs, and we confirmed observations done in general by other authors.

CONCLUSIONS

The students and teachers structured linking metaphors, usually visually, in such a way that suddendly provokes missunderstandings When they are grounding metaphors, the situation is more complicate. The use of dynamic software and the teachers decisions about their use introduces new didactical compromises. The observed presentation is characterized to create a conceptual bridge that allows certain transfusion of properties of the departure dominion towards a dominion of arrival. In other words, they create a certain " isomorphism " that allows a series of characteristics and structures to transpose, that partially organize the knowledge in the students, since it can organize a field in terms of other already known. Singular and dangerous is the fact of the metaphor is that it's compatible with the literal falsity. The following phenomenon related to the didactic uncontrolled use of this phaenomena is observed when carry out activities with computer software: "(...) we observed that there were students who, when they moved the **A** point, thought that the new point continued being the **A** point and that the new tangent straight line was the same one that before but with a different inclination. In fact, it is as if they structured the situation in terms of a person who moves (**A** point) with a bag in the back (straight tangent) by a highway that first raises and later lowers (graph) and that considered that the person and the bag were the same in spite of being in different places and having different inclination." (as it was observed by Font, 2000, pág. 122).

From the analysis of the videos, from the interviews made to the teachers and the answers to the questionnaires on the side of the students, we arrive, among others, to the following conclusion: The teacher uses not very consciously expressions that suggest, among others, metaphoric roles viewed by the idea that "the graph of a function can be considered as the outline that leaves a point wich moves subject to certain conditions -

or a variation of this metaphor: "the graph of a function can be considered as the outline that leaves a point wich moves on the graph. -. These metaphors are not innocuous and produce significant effects in the students' understanding and construction of derivative properties.

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