Designing Professional Tasks for Didactical Analysis as a research process

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In the present paper, we show a process of designing, assessing and re-designing (following DBR methodology) professional tasks for preservice mathematics teacher training for Secondary School, based on Ontosemiotic perspective for cognition and mathematical instruction (OSA) and the correspondent reflective analysis about associated professional practices. Such a process has been carried out during three consecutive years in the context of the Master's degree for training teachers of mathematics in Spain. The study shows how the successive revisions promote growing depth analysis in the teaching school practices of the future mathematics teachers.

Keywords: Professional tasks, design based research

1. Presentation

In this paper, we show a part of a wider investigation in which we analyze the design of professional tasks in the teachers' formation of Secondary Mathematics Teachers. We focus on the role of design based research (DBR) and teaching experiment (Gravemeijer, 1998) analyzing the planning cycle and redesign of our training process in successive phases, aiming the growing and building knowledge for teaching (Zavlaswski & Sullivan, 2011) by future teachers. We explicitly focus ourselves in recognizing factors that promote the feedback in the design of professional tasks for development of the of didactic analysis competencies of the future mathematics secondary teachers. Our intention is that they can develop sequences of suitable tasks and to be able to re-plan their own designs of school tasks. This work has been carried in a funded Research Project (Assessing and developing professional competencies in mathematics and didactics during initial Secondary Mathematics Teacher Training courses) being the first two authors of this work members of the team who implemented the course. The professional tasks have been evaluated and re-designed by the whole research team during the period 2009-2012.

In our study we call professional task those tasks that we propose to the future teachers in order that they realize didactic analysis and develop their didactic analysis competencies understood as the ability for designing, applying and evaluating sequences of learning by means of didactic analysis techniques and quality criteria. The aim is to establish cycles of planning, implementation, evaluation and proposals for improvement. It is also assumed that one could identify criteria and indicators regarding the development of this competence and how it relates to the other professional competencies required by future secondary school mathematics teachers. This assumption is related to the question: How might professional mathematical tasks being designed in order to make best use of the opportunities for being a teacher as teacher enquirer? (Mason & Johnston-Wilder, 2004).

Our main aim is to investigate how the process of building a sequence of professional tasks (so called formative cycle from now) promotes and generate feedback in the development of the didactic analysis competence of the future teachers within the context of teacher training courses. Such above mentioned development, it is stated when future teachers incorporate and use tools for the description, explanation and process valuation of mathematical school teacher/learning practices. By using our professional tasks design as a design based research cycle, we also want to improve a teacher as teacher-researcher of their own practice. We will show later, examples of professional tasks and reflections of the future teachers on having solved them, that have served to provoke successive feedback that have allowed to improve the sequence of tasks, and to make it increasingly effective.

2. First year design. Building a formative cycle

In the frame of the Project of investigation mentioned, we design and implement diverse formative cycles as teaching experiments for developing transversal competencies as citizenship, digital competency, didactical analysis, and others. In particular, in this presentation we discuss a part of a cycle of formation, named of "Didactic Analysis "articulated across diverse subjects of the courses. The development of the cycle has been based from the beginning in considering six big types of professional tasks:

a) Analysis of practices, objects and mathematical processes.
b) Analysis of didactic interactions, conflicts and norms.
c) Valuation of tasks and classroom episodes using criteria of didactic suitability or quality.
d) Planning and implementation of a didactic unit in their period of practices.
e) Analysis and valuation of the suitability of the didactic implemented unit.
f) Offer of a well-taken improvement of his didactic unit, for a future implementation. This proposal is realized in Master's final Work.

During the first two types of tasks (a - b) it’s expected to appear and discuss tools for a descriptive and explanatory analysis that serves to answer “what happens in the classroom and why?” (Font, Planas y Godino, 2010). The analysis and description of the mathematical activity is realized using the theoretical constructs proposed for OSA. In this perspective (Godino, Batanero y Font, 2007), mathematical activity plays a central role and is modelled in terms of systems of operative and discursive practices. From these practices the different types of related mathematical objects emerge building cognitive or epistemic configurations among them (see two internal hexagons in Figure 1). Problem-situations promote and contextualize the activity; languages (symbols, notations, and graphics) represent the other entities and serve as tools for action; arguments justify the procedures and propositions that relate the concepts. Lastly, the objects that appear in mathematical practices and those which emerge from these practices might be considered from the five facets of dual dimensions. Both the dualities and objects can be analyzed from a process-product perspective, a kind of analysis that leads us to the processes shown (decagons in Figure 1).

During the following type of tasks (c - f), we present theoretical tools (suitability criteria, according Godino, Batanero y Font (2007) for a valuated analysis serving to answer “what could we improve?” These criteria are as follows: *Epistemic suitability* refers to the extent to which the mathematics taught are ‘good mathematics’. Thus, in addition to the specific content of the curriculum the institutional mathematics on which it is based are also used as a reference. *Cognitive suitability* reflects the degree to which the teaching objectives and what is actually taught are consistent with the students’ developmental potential, as well as the match between what is eventually learnt and the original targets. *Interactional suitability* relates to the extent to which the forms of interaction enable students to identify and resolve conflicts of meaning, and promote independent learning. *Mediational suitability* refers to the availability and adequacy of the material and temporal resources required by the teaching/learning process. *Affective suitability* reflects the students’ degree of involvement (interest, motivation, etc.) in the study process. *Ecological suitability* refers to the degree of compatibility between the study process and the school’s educational policies, the curricular guidelines and the characteristics of the social context, etc.

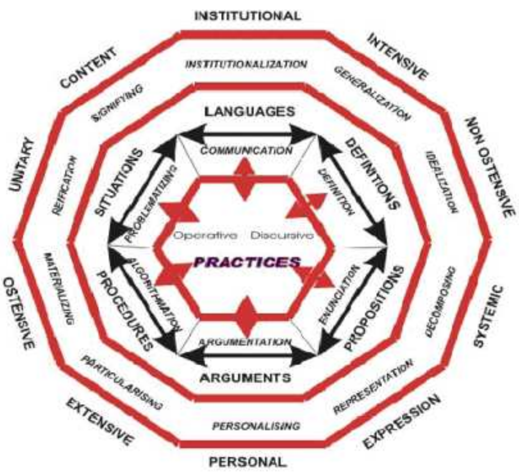


Figure 1. Onto-semiotic representation of mathematical knowledge (Godino, Batanero y Font 2007)

We understand that the study of descriptive and explanatory analysis for a didactical situation is necessary to argue based valuations (Pochulu y Font, 2011).

Methodologically, the research is mainly qualitative in nature as the purpose is to describe the development of competence in didactic analysis among aspiring secondary school mathematics teachers, from the University of Barcelona (Spain). The data were collected from the video recorded observations, sorting sheets produced by the teacher trainers and their reflections at the end of the workshops and using the documentation housed in the Moodle platform (slides, reading material, tasks and the students’ responses to them, and questionnaires and the students’ responses to them) and printed material. The samples were intentional. During all these academic years, in general, these students vary in the amount of mathematical knowledge they have, and they hold certain conceptual biases regarding the teaching and learning of mathematics.

It was selected an initial task in which students confront a short case study about proportional reasoning, using transcripts of a classroom situation. Such an initial ***tasks*** (type a; non-theoretical), introduce the students for reading and analysing the classroom example, by using their previous knowledge and beliefs of didactic analysis. During the first task, future teachers did naïf comments about a proportion class. It's easy for them to identify mathematical objects but it's difficult for them to recognize all the processes involved in the task. When they analyze interactions, they focus on leadership and teacher interventions (Task b). It's difficult for the future teachers to identify epistemic conflicts and norms. In such first analysis each group of future participant teachers used just implicitly some of the levels of analysis proposed by OSA (described in Font, Planas y Godino, 2010): Analysis of mathematical practices; Analysis of objects and mathematical processes activated by these practices; Analysis of didactical interactions and conflicts; Identification of systems of norms conditioning and making possible instructional process; Valuating didactical suitability of instruction (Font, et al., 2012). In the class debate it was observed that, though every group did not use all these levels of didactic analysis, it is possible to see how the student group as a whole has contemplated the five levels of analysis. The tasks (a-b) were considered fruitful, so they were conserved for new implementations, but they should be completed by means of the 'other voices' technique (Garuti & Boero, 2002).

Later, in the different subjects of the Master, the students realize other analysis of practices, objects activated in the above mentioned professional practices (problem, definition, proposition, representation and argument) and mathematical processes (type a). Observing the analysis realized by the future teachers some difficulties appear: (1) Difficulties to distinguish between concepts and definitions, (2) Duplicity between definitions, propositions and procedures; (3) Duplicity between propositions and thesis of arguments; (4) The description of practices is overlapped by the configuration of objects and by the description of processes, (5) Difficulties to observe and to catalogue mathematical processes, etc.

It also had been designed and implemented tasks (type b), with protocols served to show constructs as cognitive and semiotic conflicts, epistemic obstacle, types of norms, interactive, patterns of models of management, etc. After first year of experience we found that protocols were statics. For the next year it was decided to use videos and corresponding transcripts. After that it was analyzed a class about equations by applying suitability criteria (task type c). The students star by analyzing mathematical practices, objects and processes. Then the teacher develops an example in which it was revised suitability construct. After that the future teachers reflect, improve and refined their analysis by using the notion of epistemic suitability. Nevertheless, it's still difficult for the students to identify some semiotic conflicts.

Next it was proposed a task of planning and further implementation of a didactic unit in their period of practices (task type d). When doing the analysis and valuation of the didactic implemented unit (task type e), future teachers found that their planning was conditioned by the school plans in which they did the practices. As a consequence it was difficult for them to identify the epistemic consideration implicit by the school teacher proposal.

The future teachers had a few autonomy to apply in the design and implementation many learned knowledge. This aspect was considered a difficult problem to solve during redesign process because of institutional framework for the proposal, which did not deal a selection of schools. The tasks type (e) and (f) are considered activities producing the feedback for future teachers and trainers. According to task type e, it was found a superficial use of theoretical tools for valuing teaching practices, due because we had short time to discuss after school practices and a need for more discussion about suitability criteria, and analysis based upon previous experiences. This aspect should be promoted in a redesign.

Positive results and some feedback from the first year are explicitly observed (when analyzing task type e). It was decided to redesign task type a, by emphasizing the analysis of processes. Another aspect to consider when redesign is to find so enough rich episodes which serving to propose different typologies to profit a short time available, instead of using different episodes in each task. It was also observed that some of the final practices' works (task type e) and master's thesis (task type f) were found so rich to be considered as episodes to be incorporated in a later redesign processes.

3. Second year redesign. Improving process analysis

It was decided not to do important changes of the cycle itself for the second year Project. As an important example for the redesign, we consider enlarging task type (a) and (b) by using a new video source. In such a new task (type a) it was proposed the observation of three short ways of introducing perpendicular bisector with 12-13 years old students, by observing three different teachers. The main idea is to present a discussion about the different practices, objects and mathematics processes and to introduce a reflection associated to how each of these classes contribute to introduce different kind of epistemic configurations and objects (see hexagons in figure 2) associated to three different definitions.

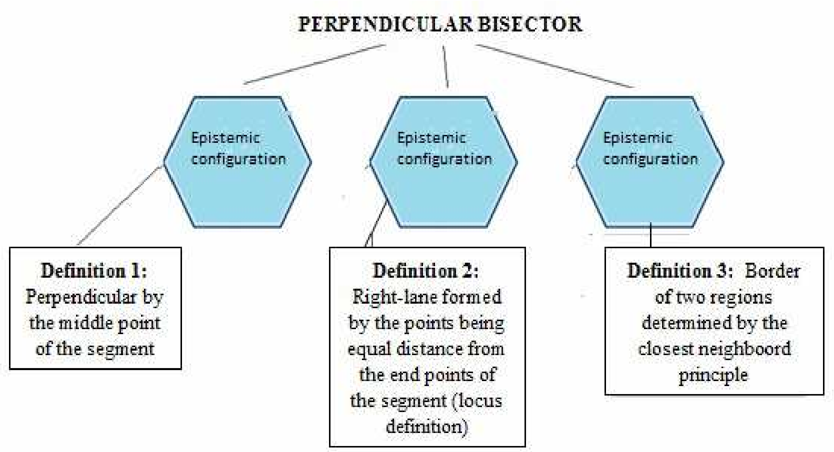


Figure 2. Epistemic Configuration of classroms for teachin g perpendicular bisector

It was observed that both first and second teachers did classical proposals and management about the content and the classroom. The third teacher proposal is innovative not only because of the management but mathematically as a way of changing the regular use of mathematical content as a change of configuration of practices, objects and mathematical processes by using a non-routine task (Tzur, Sullivan, & Zaslavsky, 2008). The class started by presenting a contextualized problem, driving to the division of a desert in a set of regions. Future teachers observed interpretation processes, communication of didactical and mathematical meanings, etc. Furthermore it appears a reflection about distinguishing complex processes from simple processes and also a general reflection about the idea of processes itself.

During the second year, the tasks designed had achieved the effect of improving their analysis of practices, objects and mathematical processes and mainly about processes (Font, et al. 2012). In this improvement, it was judged a crucial role of dynamic videotapes to analyze the visualization of professional didactical processes. On the other hand, they were introduced selected episodes of students' from previous years that were considered as a short distance from prospective teachers' perspectives. We still detect that the future teachers applied epistemic suitability criteria, by means of superficial explanations, short justifications, etc. Therefore, it's needed to improve future teachers' justifications about mathematical and didactical quality of their practices as a basis of the second redesign.

4. Third year redesign. Conectness and representativeness

Epistemic suitability criteria explained for years 1 and 2 were basically sustained in the idea of representativeness, understood as a degree, of representation of learned meanings representing relations to referenced meanings. Due to the superficiality of some students' works during the moment to apply such criteria, it was decided to do an extensive study about how the students have been applied epistemic suitability criteria in their final masters' thesis (to see if they have been used the representativeness criteria, introduced some personal proposals, etc). As a consequence, the changes proposed for the third year were the following: (1) To join the categories for epistemic suitability from OSA with categories from the quality for mathematics instruction given by Hill (2010). In such a way, it was introduced new criteria for valuing mathematical quality as it is: mathematical richness, coherence, errors, etc. (2) To select new case studies from previous years students with more wide and complex explanations than the previous case studies used en year 1 and 2. The aim was to connect echoes and voices to produce more consistent arguments (Garuti & Boero, 2002) to justify mathematical quality of didactical sequences.

A prototypical example of this new task (type c) is a case based analysis upon a student that planned a sequence with 7th grade (13-14 years old students) for Thales theorem. The main idea is to use as a new task, a voice of a previous future teacher M that analyzed her own practice about Thales Theorem after the school practice during the course 2010-2011. It was observed that M did a personal final analysis in which she said "…Additionally, we have tried to establish connections either with the concepts of the unit (relating as an example, Thales with similar triangles; similar triangles with similar figures, and so on) as with other subjects (for example, to compute the measure of a columns with mirrors, Snell's law of refraction, relating phisical concepts to mathematical concepts)... So, in conclusion...my epistemic configuration was right" (St. M; final report of practice and master's thesis, 2011).

The student M did not really a real good mathematical connection (Figure 3). We use such mistake to introduce our new professional task. In such task (type c) we presented three documents: (1) tasks proposed by M to explain Thales theorem in her proposal for school practice; (2) the analysis of epistemic suitability about M proposal, and (3) a textbook in which it was ensured the representativeness of epistemic configurations for Thales Theorem having a coherent connection (see figure 3). When doing the task it was promoted a discussion to understand the idea of representativeness (by using epistemic configurations 1 and 2) and the idea of coherent connection by using triangles in Thales position. The textbook sequence follows the order: Thales, triangles in Thales position, and after that, similarity triangles.

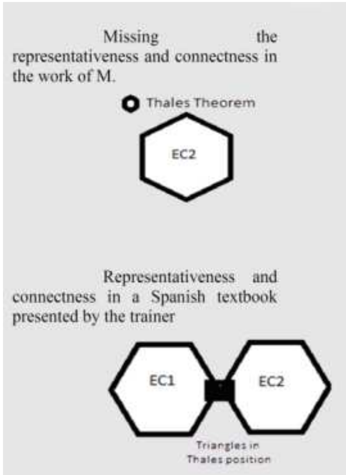
Epistemic Configuration EC1 Thales Theorem	Epistemic Configuration EC2 Similar triangles	
Concepts: point projection; segment projection; Triangles in Thales position **Properties:** **Version 1.** Segments determined by parallel rights over two secants are propotionnal. **Version 2:** Two triangles in Thales position have proportional sides and equal angles.  **Procedures** Find fourth proportion **Argument:** Comprovation of Thales Theorem; justification that triangles in Thales position have proportional sides, by using Thales theorem twice. **Problems** (1) Find one out of four segments determined by parallel rights over two secants (contextualized / decontextualized) (2) Indirect computation of unknown measures, using Thales Theorem	**Concepts/definitions:** similar triangles (similar triangles have proportional sides (def) and equal angles.) **Properties:** similarity criteria **Procedures:** Find fourth proportion **Arguments:** Comprovation of four proportional sides and equal angles	

Figure 3 Left. Two connected configurations in the textbook.
Figure 3 Right. Comparing representativeness and connectness for M and the textbook presented by the trainer.

The aim of this professional task is to recognize a deep level of analysis from such previous prospective teacher's practices (Choppin, 2011). Thus, the future teachers learn from this analysis, the idea of connecting two epistemic configurations. At the end of the third year experiment, we found the students being more carefully presenting their didactical unit as a result of such deeper analysis.

More aspects were observed in this third year, when we analyze final work of future teachers and we found better results than previous years. Just some aspects had been presented in this paper because a lack of space.

5. Conclusions. Perspectives

As a result of our study, we have analyzed in depth what we nominated professional tasks to promote competency of didactical analysis. It was useful for such analysis the levels of didactical analysis proposed by OSA. We assume the power of analyzing case studies based on texts from previous years' students. In fact, it explains the complexity of analysis that the teacher should realize to value his/her own practice to go beyond from narratives and descriptions. We are centered specially in how successive redesign contribute to have better feedback about the analysis of processes and valuing quality using notions of representativeness, connection and coherence. One of our conclusions is that to reflect about mathematical quality it's needed that the future teachers use theoretical powerful elements (Krainer, 1993). We value that some students explicit that by doing master's degree work, "we had been developed our competence of didactical analysis". On the other hand, we recognized the final master degree as the starting point for developing research competency for future teachers. In fact, it gives opportunities for students learning and recognizing problems of their professional context (Giménez, Font, Vanegas y Ferreres, 2012). Following our perspective we intend to see didactical analysis beyond the banality, considering classroom situation as an integral but dynamic system evolving in time, promoting autonomous mathematical thinking and independent validation of its results as future teacher (Laborde, Perrin-Glorian, Sierpinska, 2005).

Our major conjecture in terms of designing didactical sequences of professional tasks for prospective teachers, is that we need epistemic and cognitive analysis not only to criticize each task itself, but to adapt its connections as best as possible to the didactic analysis results. In fact, suitability criteria used for redesigning the tasks (considered as teaching experiments and corresponding case studies) has anticipatory purposes as hypothetical trajectories, but also helps to improve didactic training trajectories. It's important for our task analysis to identify difficulty factors providing frameworks for hypothesizing instructional designs inspired by levels of suitability.

The relevant aspects for our task design proposal are: (1) To understand the redesign process as a teaching experiment, assuming the noticing process (Mason & Wilder 2002) when doing didactic analysis. (2) To use suitability criteria for building and analyzing professional tasks and sequences; (3) To have in mind ethical perspectives of hearing the voice of the prospective teachers as self regulating process. (4) To consider the need for a collaborative research team for redesigning process.

After three years of experience, we assume the difficulties of the future teachers for having a deep reflection upon their proposals (Leikin, 2009) but we consider that our training cycle give opportunities to improve such issues.

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