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IMPROVING REFLECTIVE ANALYSIS OF A SECONDARY SCHOOL MATHEMATICS TEACHERS PROGRAM

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Abstract

In this paper we present how the redesign of professional tasks in the teachers' formation of Secondary Mathematics Teachers influences changes in didactical analysis competency of future secondary school teachers. We draw on data collected from 3 groups of prospective teachers, using qualitative methods. We discuss how the training on the use of didactical tools to redesign tasks led prospective teachers to further develop their own professional competence to analyse mathematical tasks from a rigorous didactical point of view.

PRESENTATION AND CONTEXT

In this paper we analyse how a specific mathematics teachers' training program may produce changes in terms of future secondary school teachers' competence of didactical analysis, aiming at the growing and building knowledge for teaching (Zavlaswski & Sullivan, 2011). Our general intention in such a program is to lead future teachers to develop the [professional] ability to (re)design sequences of suitable tasks, as well as to make them able to re-design their own designs of school tasks. In our study we call 'professional task' those tasks that we propose to future teachers in order to encourage them doing didactic analysis and developing their didactical analysis competencies. We understand 'competence' as the ability for designing, applying and evaluating sequences of learning by means of didactic analysis techniques and quality criteria. It is also assumed that someone may reflect and improve their competence in terms of the analysis of mathematical classrooms, in order to make best use of the opportunities for being a teacher as teacher enquirer (Mason & Johnston-Wilder, 2004).

We want to focus on some immediate effects over the Program. We found them when analysing prospective teachers' thoughts emerging from their feedback [work assignments] with the researchers; and also emerging from our analysis of some impacts of the program itself. Such above mentioned development, it is stated when future teachers incorporate and use tools for

the description, explanation and process valuation of mathematical school teacher/learning practices.

THEORETICAL FRAMEWORK

We introduce a teaching project based on an inquiry and reflective practicing framework in which we design and implement diverse teacher training cycles as teaching experiments (Tzur, Sullivan & Zaslavsky, 2008) for developing transversal competences as citizenship, digital competency, didactical analysis, among others. In particular, in this presentation we discuss a part of a teacher training cycle named "Didactic Analysis" which has been articulated across diverse subjects throughout the courses.

The development of the cycles had been based from the very beginning on the research process including six big types of professional tasks: (a) analysis of practices, objects and mathematical processes in which it is expected to appear and discuss tools for a descriptive and explanatory analysis that serves to answer "what happens in the classroom and why?" (Font, Planas y Godino, 2010); (b) analysis of didactic interactions, conflicts and norms; (c) evaluation of tasks and classroom episodes using criteria of didactic suitability or quality; (d) design and implementation of a lesson in their period of internship; (e) analysis and valuation of the suitability of the didactic implemented unit; (f) improvement of their lessons designs (for future implementation), within the Master's Final Project (MFP).

The analysis and description of the mathematical activity is conducted using the theoretical constructs proposed by the 'Ontosemiotic' approach (OSA). According to this perspective (Godino, Batanero y Font, 2007), the mathematical activity plays a central role and it is modelled in terms of systems of operative and discursive practices. From these practices the different types of related mathematical objects emerge building cognitive or epistemic configurations among them. Problem-situations promote and contextualize the activity; languages (symbols, notations, and graphics) represent the other entities and serve as tools for action; arguments justify the procedures and propositions that relate the concepts. Lastly, the objects that appear in mathematical practices and those which emerge from these practices might be considered from the five facets of dual dimensions. Both the dualities and objects can be analysed from a process-product perspective, a kind of analysis that lead us to the processes shown. During the following type of tasks (c - f), we present theoretical tools (suitability criteria, according Godino, Batanero and Font (2007) to conduct evaluative analysis to answer "what could we improve?" We understand that the study of descriptive and explanatory analysis for a didactical situation is necessary to justify the evaluations (Pochulu & Font, 2011).

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METHODOLOGY

The research is mainly qualitative in nature as the purpose is to describe the development of competence in didactic analysis among aspiring secondary school mathematics teachers, from the University of Barcelona (Spain) during the Project development (2010-2013). The data was collected from video recorded observations, sorting sheets produced by teacher trainers, students' reflections at the end of the workshops and documentation housed in the Moodle platform (slides, reading material, tasks and the students' responses to them, and questionnaires and the students' responses to them). The samples were 3 groups of 24-26 and 25 prospective teachers. This amount of teachers includes almost the totality of students recruited in the Teacher Program in the University. During all these academic years, in general, these students vary in the amount of mathematical knowledge they have, while discussing certain conceptual biases regarding the teaching and learning of mathematics.

During the first year, future teachers did many naïf comments regarding the first tasks (a-b). We conjectured that protocols were static. During the next year we decided to use more videos and transcripts than during the previous one. Prospective teachers designed and implemented tasks (type b), with protocols showing constructs as cognitive and semiotic conflicts, epistemic obstacles, types of norms, patterns of models of management, interaction analysis, and so on. After that, they analysed a lesson focused on equations applying suitability criteria (task type c). Future teachers reflected, improved and refined their analysis by using the notion of 'epistemic suitability' (Font, Planas y Godino, 2010). Nevertheless, observing future teachers' writings, it was still difficult for the students to identify some semiotic mathematical conflicts. Next we proposed them [the prospective teachers] to develop a task of planning and implementing of a lesson in their internship (task type d). When doing the analysis and evaluation of the lesson implemented (task type e), future teachers found that their planning was conditioned by the school plans in which they did their internship. As a consequence it was difficult for them to identify the epistemic consideration implicit in the schoolteacher proposal. We observed that the students focused more on the dialogue than on the mathematics involved in he lesson. For instance, Student 12 said, "*short challenges appear, with follow up questions in order to engage students in brief conversations just to clarify responses*", and many others as Student 6, talked about "*the teacher remains vigilant in order to ensure that classmates did not distract students*." The future teachers had little autonomy to apply their designed lessons. This aspect was considered a difficult problem to solve during redesign process because of institutional

framework for the proposal. The tasks type (e) and (f) are considered activities driving the feedback for future teachers and trainers.

During the second year we decided to implement some tasks type (a), by emphasizing the analysis of processes; and tasks type (b) by using new video sources. In the new tasks (type a) we proposed the observation of three short ways of introducing perpendicular bisector with 12-13 years old students, by observing three different teachers. The main purpose was to present a discussion about the different practices, objects and mathematics processes and to introduce a reflection associated to how each of these classes contributes to introduce different kind of epistemic configurations and objects associated to three different definitions. It was also introduced enough rich episodes which serving to propose different typologies to profit a short time available, instead of using different episodes in each task. It was also observed that some of the final internship reports (task type e) and master's thesis (task type f) were found so rich to be considered as episodes to be incorporated in a later redesign process.

After the second year of experience observing the analysis realized by the future teachers, some difficulties still appear: (1) difficulties to distinguish between concepts and definitions; (2) duplicity between definitions, propositions and procedures; (3) duplicity between propositions and thesis of arguments; (4) the description of practices is overlapped by the configuration of objects and by the description of processes; and (5) difficulties to observe and to catalogue mathematical processes; among others.

As a consequence, the changes proposed for the third year were the following ones: (1) to join the categories for epistemic suitability from OSA with categories from the quality for mathematics instruction given by Hill (2010). In such way, it was introduced new criteria for valuing mathematical quality as it is: mathematical richness, coherence, errors, etc.; (2) to select new case studies from previous years students with more wide and complex explanations than the previous case studies used en year 1 and 2. The aim was to connect echoes and voices to produce more consistent arguments (Garuti & Boero, 2002) when justifying mathematical quality of didactical sequences. We proposed to analyse a lesson presenting a contextualized problem, driving to the division of a desert in a set of regions. Within the works presented by the future teachers we observed interpretation processes, communication of didactical and mathematical meanings, etc. Furthermore it appears a reflection about distinguishing complex processes from simple processes and also a general reflection about the idea of processes itself. During the analysis it was observed that both first and second teachers did classical proposals and management

about the content and the classroom. The tasks designed had achieved the effect of improving prospective teachers' analysis of practices, objects and mathematical processes and mainly about processes (Font, et al. 2012). In this improvement, it was judged a crucial role of dynamic videotapes to analyse the visualization of professional didactical processes. On the other hand, they were introduced selected episodes of students' from previous years that were considered as a short distance from prospective teachers' perspectives. We still detected that the future teachers applied epistemic suitability criteria, by means of superficial explanations, short justifications, etc. Therefore, it's needed to improve future teachers' justifications about mathematical and didactical quality of their practices as a basis of the second redesign. Epistemic suitability criteria explained for years 1 and 2 were basically sustained in the idea of representativeness, understood as a degree, of representation of learned meanings representing relations to referenced meanings. Due to the superficiality of some students' works during the moment to apply such criteria, it was decided to do an extensive study about how the students have been applied epistemic suitability criteria in their final masters' thesis (to see if they have been used the representativeness criteria, introduced some personal proposals, etc.).

A prototypical example of this new task (type c) was a case based analysis upon a student that planned a sequence with 7th grade (13-14 years old students) for Thales theorem. The main idea was to use the voice of a previous future teacher M that analysed her own practice about Thales Theorem after the school practice during the course 2011-2012 as a new task. We observed that M did a personal final analysis in which she said *"...Additionally, we have tried to establish connections either with the concepts of the unit (relating as an example, Thales with similar triangles; similar triangles with similar figures, and so on) as with other subjects (for example, to compute the measure of a columns with mirrors, Snell's law of refraction, relating physical concepts to mathematical concepts)... So, in conclusion...my epistemic configuration was right*" (St. M; final report of practice and master's thesis, 2011). Some previous examples done by prospective teachers were also introduced as a new tasks (type c) by reflecting about the role of connections, drawing on three documents: (1) tasks proposed by M to explain Thales theorem in her proposal for school practice; (2) the analysis of epistemic suitability about M proposal, and (3) a textbook in which it was ensured the representativeness of epistemic configurations for Thales Theorem having a coherent connection. When doing the task it was promoted a discussion to understand the idea of representativeness and the idea of coherent connection by using triangles in Thales position. The aim of this professional task was to recognize a deep

level of analysis from such previous prospective teacher's practices (Choppin, 2011). Thus, the future teachers learn from this analysis, the idea of connecting two epistemic configurations.

SOME RESULTS ABOUT REFLECTIVE ANALYSIS

After first year observations, we found that some future teachers had difficulties to connect didactical analysis to epistemic ideas. For instance, Student 5 claims: "*When I did the didactic unit I didn't contextualized enough the exercises. Now, I think it's important to use activities proposed in the article: 'Algebra for all Junior High School students'.* In these kinds of sentences, we expected to talk more about the specific iterative algebraic approach as an explicit content in the article explained by the student. However, student 5 declined to focus his comment under such approach, and he highlighted the importance of contextualization for designing unit lessons. It's an example of the initial difficulties to accept the role of epistemic and cognitive analysis.

At the end of the third year, we found that students being to present their lesson more carefully, as result of such deeper analysis.
When analysing the final work of those future teachers we found better results than previous years. Here we'll see an example of growing ecological suitability relations among institutional framework, cognitive suitability and epistemic understanding in which the future teacher interpret why his new proposal really improves mathematical meanings. It's the case of a future teacher X (belonging to the 2012 case study group). Let's observe his explanations emerging from his written work compiled after his internship.

> The regular teacher gives to me the opportunity to improve some aspects of the classroom situation, by introducing hypermedia tools with a group of students with mathematical difficulties in another group (not the same as I did my first practice)… Therefore, I proposed "changes" in my initial proposal. In order to achieve the challenge (of expanding enriching, and consolidating the zone of personal geometrical meanings) we devoted more than a half of time during the first session to revise their previous mathematical knowledge, and also the techniques, tools, resources and operational knowledge needed (as surfaces formulae, volumes, and so on). My strategy was to create a debate among the students... by using an email-forum in which I could adjust mathematical rhythm to each student. Let's see the dialogue showing the impact of my strategy: For teacher X- Calculate the volume ... Student O- Please, X, I have a doubt, As I must calculate the basis surface, do I need to multiply twice, because I have two basis? ...
> ... I never heard about such difficulties, because I did group discussion in my first experience. Now, the one-to-one discussion provides the possibility to hear from the students... (Master's work of a future teacher X).

CONCLUSION

As a result of our study, we have analysed in depth what we call professional tasks to promote growing competency of didactical analysis year by year in our Program, considering different students. The levels of didactical analysis proposed by OSA were very useful to illuminate this didactical analysis. We assume the methodological potential of analysing case studies based on the texts coming from the students' works done in previous years. In fact, these practices may explain the complexity of the analysis that teachers should conduct to value his/her own practice to move beyond from narratives and descriptions. We found the importance of some didactical notions as representativeness, connection and coherence. One of our conclusions for prospective teachers enrolled in teacher training programs is the necessity to use theoretical powerful tools to lead them reflect on the mathematical quality of task-design or lesson design (Krainer, 1993).

After three years of experience, we found many evidence suggesting that students really transform their attitude towards using a "didactical approach" to inform their [future] professional work as teachers: "*we had been developed our competence of didactical analysis*". On the other hand, we recognized the final master degree as the starting point for developing research competency for future teachers. In fact, it gives opportunities for students to learn and recognize problems of their professional context. Following our perspective we intend to see "didactical analysis" beyond the banality, considering classroom situations as an integral and dynamic system evolving in time, promoting autonomous mathematical thinking and independent validation of results as future teachers (Laborde, Perrin-Glorian, Sierpinska, 2005). We found that the "suitability criteria" used for redesigning the tasks (considered as teaching experiments and case studies) has anticipatory purposes as hypothetical trajectories, but also helps to improve didactic training trajectories.

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