# MATHEMATICAL PRACTICES, SEMIOTIC CONFLICTS, AND SOCIO-MATHEMATICAL NORMS

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*We adapt the onto-semiotic approach to research in mathematics education developed by Godino and his colleagues (e.g., Font, D'Amore & Godino, 2007; Godino, Batanero, & Roa, 2005) in order to better understand certain disparities in the interpretation of classroom socio-mathematical norms (Yackel & Cobb, 1996). In this approach, the experience of disparities in the interpretation of norms can be conceptualized as the experience of semiotic conflicts. We explore semiotic conflicts in the mathematics classroom in relation to: 1) mathematical practices that are being developed within the classroom, 2) socio-mathematical norms that intervene in the orchestration of the practices, and 3) differences in the interpretation of the norms.*

## INTRODUCTION

We build on three theoretical concepts -mathematical practices, semiotic conflicts, and socio-mathematical norms- by looking at relationships among them in the context of the mathematics classroom. We begin with a focus on the notion of mathematical practices as viewed by the onto-semiotic approach to research in mathematics education. We then move to considerations on how this approach helps develop a joint exploration of notions traditionally drawn from different theoretical frameworks, such as semiotic conflicts and socio-mathematical norms. Finally, we present some findings from the analysis of a classroom episode where participants differ in their understanding of certain norms. We argue that the emergence of a semiotic conflict in that episode and its resolution are related to 1) mathematical practices that are being developed within the classroom context, 2) socio-mathematical norms that intervene in the orchestration of the practices, and 3) differences in the interpretation of the norms.

## MATHEMATICAL PRACTICES

The onto-semiotic approach to research in mathematics education (e.g. Font, D'Amore & Godino, 2007; Godino, Batanero, & Roa, 2005; Godino, Batanero, & Font, 2007) tackles the problem of meaning and the representation of knowledge by elaborating a mathematical ontology based on anthropological, semiotic and socio-cultural frameworks. Mathematical knowledge is considered to depend on the cultural institutions and the social contexts where the learner is implied, and on the activities s/he develops there. The mathematical activity plays a central role and is modeled in terms of systems of mathematical practices. Godino and Batanero (1994), in reference to the notion of mathematical practice, write:

> We consider *mathematical practice* [sic] any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the

solution to other people, so as to validate and generalize that solution to other contexts and problems (Godino & Batanero, 1994, p. 182, quoted in Godino, Batanero, & Font, 2007, p. 129).

By focusing on the notion of mathematical practice, the onto-semiotic approach puts the emphasis on the mathematical objects and processes that make this practice possible. Systems of mathematical practices are conformed by different types of mathematical objects (language, arguments, concepts, propositions, procedures, problems…). Here, the notion of meaning is defined in terms of the practices. For instance, the meaning of "equation" can be viewed as the systems of practices where "equations" are required or carried out. The analysis of the systems of practices that lead to the emergence of mathematical objects and processes is, therefore, crucial when trying to infer the meanings for these objects and processes.

The objects that appear in mathematical practices and those emerging from these practices take into account different "dimensions." In this report, we are especially interested in the personal/institutional dimension that is, in turn, related to different types of mathematical processes. The processes of solving problems or those of modeling are "mega-processes" than can be divided into more primary processes such as those of personalizing and institutionalizing. In the onto-semiotic approach, teaching involves the participation of students in a context of practices where processes of institutionalizing are being developed. Learning is conceived as the students' personalizing of practices that are (to be) institutionalized.

## SEMIOTIC CONFLICTS AND SOCIO-MATHEMATICAL NORMS

The role of the practices is also emphasized by the socio-cultural approach (Stephan, Cobb, & Gravemeijer, 2003). This approach, however, has given priority to other types of processes such as processes of valorization that regulate the access of the learner to the practices in the mathematics classroom. Processes of personalizing and institutionalizing have been mainly explored from the perspective of the personal and the legitimate interpretations given to norms of the mathematical practice. Godino, Batanero and Font (2007) say that "[the notion of mathematical practice] might allow a coherent articulation with other theoretical frameworks [different from the onto-semiotic framework], such as ... [the] socio-cultural approach to mathematical meaning and cognition." (p. 134).

We attempt to articulate the notion of socio-mathematical norm with that of semiotic conflict. A semiotic conflict is "any disparity or difference of interpretation between the meanings ascribed to an expression by two subjects, being either persons or institutions" (Godino, Batanero, & Font, 2007, p. 133). We refer to socio-mathematical norms as the regulations that influence participation within the mathematics classroom, and the interactive structure of the development of its practices (Yackel , & Cobb, 1996).

We view the diversity of interpretations of socio-mathematical norms from the perspective of the diversity of practices where these norms intervene. In particular, a

norm is a type of object that is especially related to processes of institutionalizing and personalizing. Differences in the interpretation of socio-mathematical norms turn into differences in mathematical practices that intervene in processes of institutionalizing and personalizing. And vice versa, differences in the mathematical practices that intervene in these processes turn into differences in the interpretation of norms.

The experience of differences in the interpretation of norms can be conceptualized as the experience of semiotic conflicts. For instance, the different understanding of a norm such as "working collaboratively when solving a problem" constitutes itself a semiotic conflict when subjects differ in their interpretation of the word "collaboratively" -and therefore carry out different practices in their collaboration with other subjects-, or when they differ in their interpretation of what counts as "communicating when working collaboratively."

The interpretations of norms are constructed under the influence of at least three issues: 1) the participation in a diversity of contexts of institutional knowledge, 2) the reconstruction of personal expectations, values and other norms, and 3) the interaction with others. The focus on each of these issues leads to three main types of semiotic conflicts: epistemic, cognitive and interactional. Any conflict is to some extent and at the same time, epistemic, cognitive and interactional. A girl may have constructed an interpretation of the expression "working collaboratively" during her participation in a girl scouts group; this may differ from that given by the teacher in her mathematics classroom, and also with that given by her personal reconstructions as a result of knowing the peers with whom she is supposed to "work collaboratively."

The institutional meaning given to "subtraction algorithm" by a group of Moroccan immigrant students, in a mainstream school where the group of local students has been taught a different algorithm, is an example of epistemic conflict. The mathematical object "subtraction algorithm" can be used differently depending on the contexts of institutional knowledge that have had an influence on the students of each group. This epistemic conflict is linked to the interpretation of the socio-mathematical norm that regulates the subtraction procedure to be used in the classroom. We briefly analyze another example of epistemic conflict in the next section.

Certain contradictory practices can point to a cognitive conflict. A student may properly operate with fractions, but after having studied the equations -where denominators are deleted at a certain stage of the process of solving equations-, s/he may transfer the procedure with equations to operations with fractions. The socio-mathematical norms that regulate operations with fractions and equations are involved in the emergence of this conflict, though there may be other norms involved such as "teaching equations without considering the fractions that 'constitute' an equation."

Interactional conflicts can occur in situations of exchange such as student-student or student-teacher, when participants differ in the interpretation of an expression. A

teacher in a classroom may draw a non-prototypical triangle and refer to it with the word "triangle", but a student may not recognize the drawing as a triangle because s/he is only used to visualize prototypical examples. What counts as the representation for a triangle -which it itself a socio-mathematical norm- is differently understood by the student and the teacher. Another student's interpretation of the object "triangle" can be based on the visualization of portions of pizza or traffic signals, which are objects from an "out-of-school" epistemology. The next example refers to an epistemic conflict that is also interactional as it is mainly expressed through confronted linguistic manifestations in the classroom context.

## AN EXAMPLE

We present an example of semiotic conflict and part of its interpretive analysis by drawing on data from a classroom episode in a high school (15 and 16 years old students). It is the first lesson where the teacher proposes a problem solving dynamics with small-group work during the whole class. The problem is about two well-known neighborhoods near the school site (see Figure 1), whose actual names were given to the students. The episode is centered on the discussion of a particular aspect of the arithmetical task. It starts during the first part of the whole group discussion, when the members of a group, Alicia (A), Emilio (E) and Mateo (M), tell the teacher (T) that they have developed different approaches to the task. The episode finishes when the teacher shifts from exploring this group's ideas to attempting to make other groups participate. We reproduce part of the transcript, where the semiotic conflict can be inferred. The mathematical practices and the socio-mathematical norms that help understand the conflict are taken from the whole episode's transcript.

| Here you have the population and area of two neighborhoods in your town. | |
| --- | --- |
| Neighborhood 1 (N1) | Neighborhood 2 (N2) |
| 65,075 inhabitants | 190,030 inhabitants |
| 7 km$^2$ | 5 km$^2$ |

(i)     Discuss in which of these places people live more spaciously.
(ii)    Find how many people should move from one neighborhood to the other in order to live in both of them spaciously.

Figure 1. The statement of the problem

## The conflict

[1]   A: This is a problem about densities because data are about densities.

[2]   T: OK. Tell Alicia that she needs to explain herself better. [To Alicia] We know that you know a lot, but…

[3]   A: In N1 the density is lower than in N2. That's all.

[4]   T: Emilio says no.

[5]  E: I just don't see it! There's something missing.

[6]  T: [To Emilio] How have you done it?

[7]  E: It's clear that here [N2] there are more people and less space. I've been there. Flats are very small.

[8]  T: OK. You say it's clear, but then how do you answer the second question? You cannot say again that flats are very small.

[9]  E: The second question is wrong.

[10] T: Why?

[11] E: I wouldn't move alone, I'd take all my family.

[12] T: What do you mean?

[13] E: I would change the second question.

[14] T: Don't start again, Emilio! You know that problems are like they are.

The semiotic conflict occurs when the teacher tells Emilio that the reasoning used when answering (i) will not be useful when trying to answer (ii) [8]. For Emilio, the first question of the problem is rather obvious, given his knowledge of the neighborhoods. The teacher's comment does not make him change his approach. Instead of developing a "more mathematical" reasoning or a reasoning less based on his knowledge of the neighborhoods, and despite the teacher's requirement, the student maintains an argument based on his "out-of-school" knowledge [11]. During the group work (which is not part of the transcript we discuss), Emilio had made similar comments showing him connecting the problem to his "out-of-school" knowledge.

Later the teacher centers his efforts on the process of institutionalizing Alicia's approach to the problem. This student introduces almost all the mathematical practices during the episode by properly using symbolic and verbal mathematical languages, constructing arguments ("This is a problem about densities", "In N1 the density is lower than in N2", "9,296 is smaller than this number [38,006]…), referring to mathematical concepts ("fractions", "equations", "decimals"…), introducing mathematical procedures ("dividing", "comparing densities", "solving equations"…), and establishing relationships among these objects in various propositions. Most of Alicia's mathematical practices are represented on the blackboard, such as that of changing 65,075 to 65,072 [23] (see transcript below). She removes 3 people from the original number of people because she wants the property of working with a multiple of 7 and not having decimal numbers.

Alicia also develops the practice of solving "contextualized" mathematical problems. The teacher points to the importance of the process of going back and forth between the "mathematical context" and the "real context" when solving a problem whose statement is based on "real life." The situation in which the problem happens and that helps to understand its solution -the "real context"- must be referred to at certain stages of the solving process. However, Emilio does not accept moving from his knowledge of the neighborhoods to the mathematical knowledge that he is expected

to have. Alicia shows more flexibility. After having listened to one of the teacher's comments [24], she moves to the "real context" suggested by the statement of the problem, and later she goes back to the "mathematical context" again.

During the episode, two socio-mathematical norms explicitly regulate the practice of solving "contextualized" mathematical problems:

*Norm 1*: At the end of the solving process, the solution must be interpreted within the real context suggested by the problem.

[23] *A*: [On the blackboard]

$$\frac{65,075}{7} \rightarrow \frac{65,072}{7} = 9,296 \text{ h/km}^2 \text{ at N1}$$

$$\frac{190,030}{5} = 38,006 \text{ h/km}^2 \text{ at N2}; 9,296 < 38,006$$

[24] T: OK. We need to compare the two neighborhoods. These numbers mean nothing if we don't compare them.

[25] A: This number [9,296] is…

[26] E: We place some people here and some people there.

[27] A: Let me finish! 9,296 is smaller than this number [38,006]. This means that in N1 you live more spaciously.

[28] T: OK.

*Norm 2*: At a certain stage of the solving process, the mathematical context must prevail over the real context.

[38] T: Mateo, let's concentrate, now you forget about the people and you only think about the fraction. Is 65,075 a multiple of 7?

[39] M: No.

[40] T: That's the point! 65,072 is a multiple of 7 and 65,075 isn't. Now it can be divided exactly.

[41] M: But it's not about multiples, it's about people!

Emilio does not adjust to these norms, neither during the experience of the conflict nor during the teacher's comments that follow this experience. By using his "out-of-school" knowledge, Emilio resorts to his understanding of an appropriate way of solving "contextualized" problems in the mathematics classroom. He insists on interpreting the solving process in the real context that is suggested by the statement of the problem. Alicia suggests a process of generalization when she repeatedly considers that the problem is a particular case of a certain type of problems -problems about density- [1, 3, and later in the transcript]. Emilio, however, refuses to accept this argument. He also refuses the process of idealization that the teacher wants Mateo to carry out when he wants this student to focus on the fraction and its representation on the blackboard [38].

We interpret that the conflict has to do with how to determine the valid methods of solving "contextualized" mathematical problems when they seem to be different

whether being applied in a real life context or in the institutional context of the mathematics classroom [9-13]. Although the teacher points to an important meta-epistemic norm of the mathematics classroom [14], makes two socio-mathematical norms explicit, and leads Alicia to develop rather good mathematical practices, Emilio insists for a while on using his "out-of-school" knowledge and shows a certain resistance towards the institutional meanings defended by the teacher. At the end of the episode, Emilio finally accepts copying the equation in his notebook and gives up on his attempts to change the approach to the problem for the rest of the session. Here, the resolution of the epistemic conflict must be understood as the public act of not insisting on the importance of the "out-of-school" knowledge anymore.

## FINAL REMARKS

Some of the teacher's initial efforts when trying to make Emilio experience a cognitive conflictive [8] do not seem to be directly related to this student's sudden change in his positioning. Despite the identification of some possible issues of influence for this change to occur, how firmly can we affirm that Emilio's experience of a conflict has been solved? Is it possible to modify a positioning concerning which knowledge is appropriate in a mathematics classroom only by listening to a few norms and paying attention to some good mathematical processes? What if changes in Emilio's discourse seek to improve his relationship with Alicia and with the teacher, and do not express conviction? Is Emilio publicly acknowledging practices that he does not own? The onto-semiotic approach helps explore experiences of semiotic conflicts and issues of influence on the public resolution of these conflicts from the perspective of the practices and the norms involved. But we still need to better understand how learners experience others' interpretations of socio-mathematical norms and mathematical practices.

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